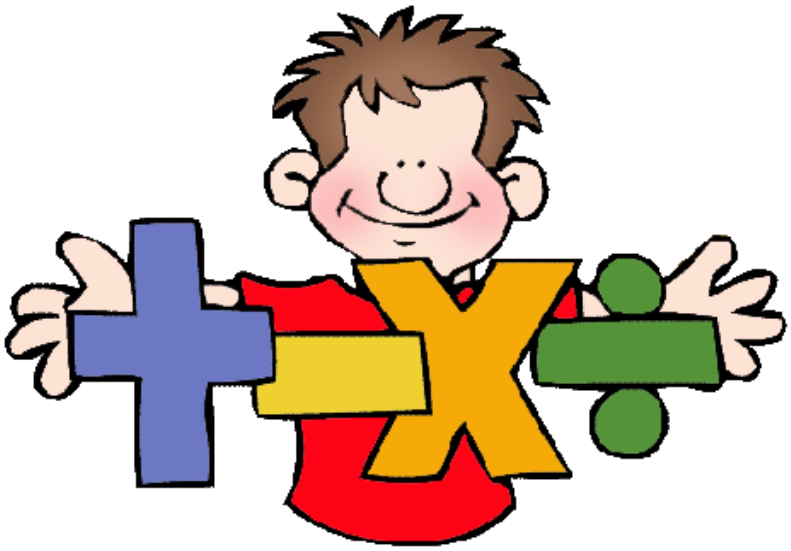
Mathematics

2 ESO



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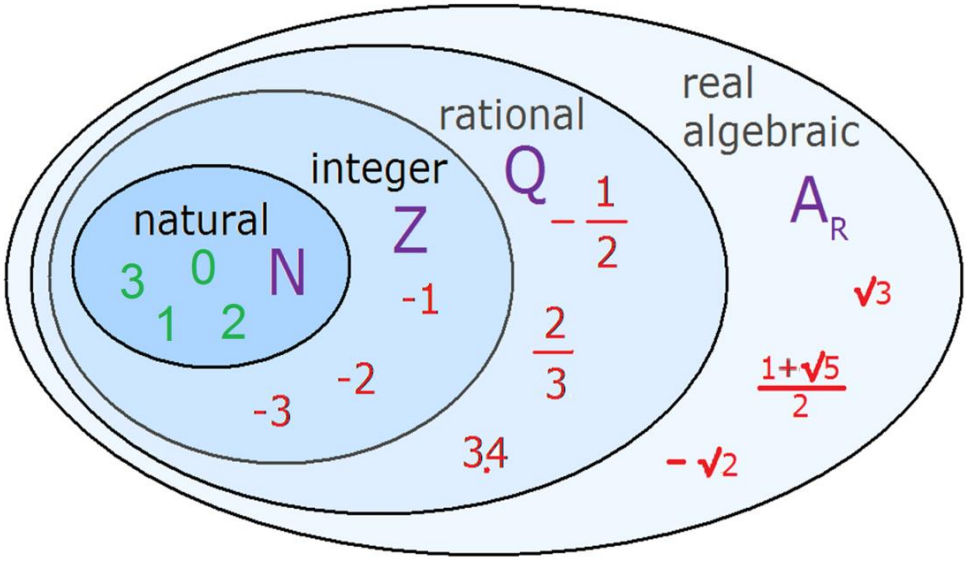
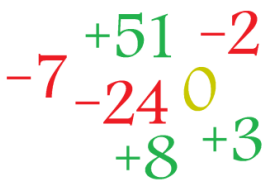
Chapter 1

INTEGERS





Remember: Integers contain the natural numbers, the negative numbers, and zero.

We can represent integers on a number line.

1 Draw a number line and on it indicate the position of numbers: –1, 5, –4 and 7.

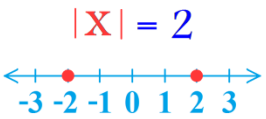
2 In the previous activity, which of the four numbers is:

a) Most to the left? Why?

b) Most to the right? Why?

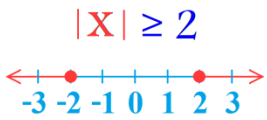
3 Draw another number line and mark on it these temperatures: 18°, 0°, –2°, –7°, 9°.

1.1 Absolute value



The "absolute value" of an integer is the numerical value without the negative or positive sign.

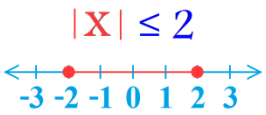
On a number line, the "absolute value" is the distance between the number and zero.



The absolute value of –2 is 2. The absolute value of +2 is 2.

The symbol for absolute value is the number between vertical bars:

|–150| = 150 "The absolute value of –150 equals 150".



1.2 Adding and subtracting integers

1st Add the positive integers.

2nd Add the negative integers.

➢ The result will have the sign of the integer with greater absolute value.

3rd Subtract the number with a smaller absolute value from the number with a greater absolute value.

4th Give the result the sign of the integer that had greater absolute value.

Add: –3 + 5 + 4 – 5 – 6

1st we add the positive integers:

+5 +4 = 9

2nd we add the negative integers:

-3 - 5 - 6 = -14

3rd wesubtract |9| from |-14|:

|-14| - |9| = |5|

4th the result has the sign of the integer with greater absolute value:

|-14| > |9| so the sign is: (-).

Result: -5

4 What is the absolute value:

a) –8 c) 8 e) 89

b) 3 d) –4 f) –153

5 Add and subtract these integers:

a) 6 – 12 + 6

b) 8 – 20 + 34 – 2

c) –42 – 6 + 36

d) –5 + 4 – 6 – 4 – 1

e) –5 – 52 – 8 + 98 – 58

f) 1 + 60 – 10 – 4 – 9 + 3

g) –2 + 1 – 62 + 8 – 1 – 8 + 7

h) 2 – 1 – 56 + 8 + 64 – 9

6 What is the altitude difference between a bird at 26 m and a fish at –9 m? And if the fish goes 4 m lower?

1.3 Multiplication and division of integers

We multiply or divide the absolute value of the integers.

The sign of the result is:

(+) · (+) or (+) / (+) → (+)	4 · 2 = **8**	4 / 2 = **2**
(–) · (+) or (–) / (+) → (–)	–4 · 2 = **–8**	4 / –2 = **–2**
(+) · (–) or (+) / (–) → (–)	4 · –2 = **–8**	–4 / 2 = **–2**
(–) · (–) or (–) / (–) → (+)	–4 · –2 = **8**	–4 / –2 = **2**

➢ Multiply these integers: 5 · (–4)

First, we multiply the absolute values: $|5| \cdot |-4| = 20$

Second, we put the sign to the result (+) · (–) → (–), so the result is: **–20**

➢ Divide these integers 5 / (–4)

First, we divide the absolute values: $|5| / |-4| = 1.2$

Second, we put the sign to the result (+) / (–) → (–), so the result is: **–1.2**

7 Multiply these integers:

a) 6 · (–2)	e) (–7) · (–4)	i) 65 · (–86)
b) 3 · (–5)	f) 1 · 5	j) 28 · 144
c) (–7) · 2	g) 1 · (–5)	k) (–37) · (–46)
d) (–7) · (–2)	h) (–2) · 2	l) (–468) · (–5)

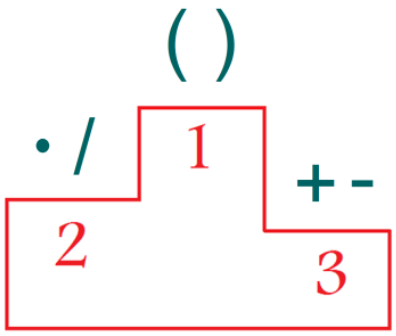
8 Divide these integers:

a) 4 / (–2)	e) (–2) / 2	i) 80 / 5
b) 10 / (–5)	f) (–2) / (–2)	j) (–80) / (–20)
c) (–8) / (–2)	g) (–10) / 2	k) 128 / 64
d) 12 / (–3)	h) (–7) / (–4)	l) 512 / (–256)

9 A plane flying at 11,000 metres starts descending at 500 metres/minute. What is its altitude after 10 minutes? And after 22 minutes?

10 www.esobook.es/b1

1.4 Order of operations



Sometimes there is a combination of operations; in that case, follow this order:

1st Operations inside brackets

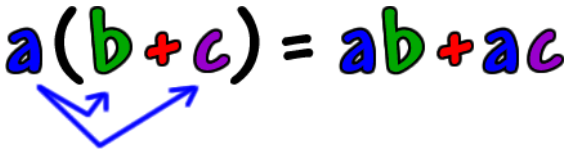
2nd Multiplication and division

3rd Addition and subtraction

Distributive property

To multiply a number by the sum of others is the same thing as multiplying the number by each of the numbers in the addition.

$$a \cdot (b + c) = a \cdot b + a \cdot c$$



Solve $1 + 5 \cdot (2 + 6)$

$= 1 + 5 \cdot (8)$

$= 1 + 40 = 41$

Check the distributive property:

a) $7 \cdot (2 + 3)$

$= 7 \cdot 5 = 35$

$7 \cdot 2 + 7 \cdot 3$

$= 14 + 21 = 35$

b) $8 + [(4 + 3) \cdot 4]$

$= 8 + (7 \cdot 4)$

$= 8 + 28 = 36$

$8 + 4 \cdot 4 + 3 \cdot 4$

$= 8 + 16 + 12 = 36$

11 Which type of operations is first in the order of operations?

12 Which type of operations is second in the order of operations?

13 Which type of operations is third in the order of operations?

14 Solve:

a) $1 + 5 \cdot (2 + 6)$

b) $7 \cdot 5 - 7 \cdot (4 + 1)$

c) $6 - 2 \cdot (6 + 1) \cdot (4 \cdot 3)$

d) $9 \cdot 5 \cdot 1 - (20 \cdot 5)$

15 Solve:

a) $2 + [(5 + 2) \cdot 4] \cdot (8 + 2)$

b) $4 + 2 \cdot [(3 + 1) \cdot (2 \cdot 5)]$

c) $[(1 + 2) / 3] / (8 - 2)$

d) $5 - (5 - 6) \cdot 5 + 3 \cdot [2 \cdot 8 / (3 + 1)]$

16 Check the distributive property:

a) $2\,(7 - 3)$ b) $-4\,(1 + 3)$

1.5 Factors

The factors of a number are the numbers that divide exactly into it, including 1 and the number itself.

For example: 12 divides exactly into 3 (12/3 = 4) so 3 is a factor of 12.

A prime number is a number that only has two factors: 1 and itself.

The number 1 is not a prime number by definition.

A composite number is a number that has more than two factors.

126	2
63	3
21	3
7	7
1	

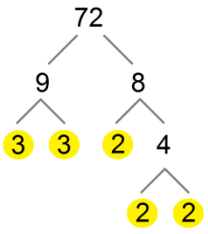
To find the prime factors of a composite number:

1st Draw a line next to the composite number.

2nd Divide the number by the smallest possible prime number and write that prime number to the right of the line.

3rd Write the quotient to the left of the line and divide the quotient by the smallest possible prime number.

4th Repeat the 3rd step until the quotient is 0 or 1.



If one of the prime factors is repeated, we can write it to the power of "how many times it is repeated".

$72 = 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 = 3^2 \cdot 2^3$

➤ Which are prime and which are composite numbers?

a) 20

20 is a **composite** number because we can divide it by 10, for example.

b) 31

31 is a **prime** number because we can only divide it by 1 and by 31.

➤ Find the prime factors of 6 and 63:

$6 = 2 \cdot 3$

2 and **3** are the prime factors of 6

$63 = 3 \cdot 3 \cdot 7$

3 and **7** are the prime factors of 63

17 Which are prime and which are composite numbers?

a) 10 d) 5
b) 17 e) 33
c) 47 f) 110

18 Which of these numbers are factors of 36?

4, 5, 6, 8, 10, 15, 18, 20

19 Find the prime factors of:

a) 20 d) 84
b) 28 e) 33
c) 160 f) 980

20 We have 132 apples. Can we put them all in bags of 12 apples? Why?

Highest common factor (HCF)

The factors of 18 are:

1, 2, 3, 6, 9 and 18

The factors of 24 are:

1, 2, 3, 4, 6, 8, 12 and 24

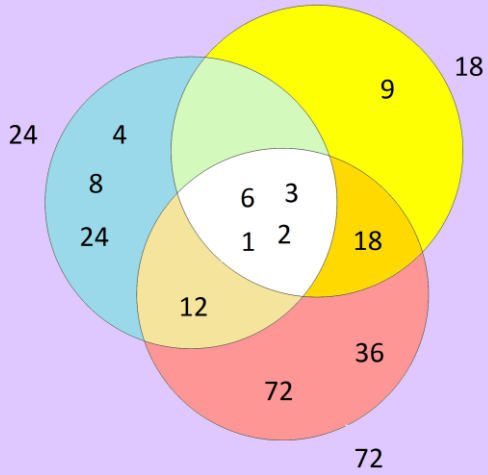
And the factors of 72 are:

1, 2, 3, 6, 12, 18 and 72

Numbers 1, 2, 3 and 6 are on the three lists, they are the common factors of 18, 24 and 72.

Number 6 is the highest number that is on the three lists, so it is the "highest common factor" (HCF).

The factors in all groups are the common factors.



The highest number in all groups is 6, so 6 is the highest common factor.

Find the highest common factor of: 24, 72, and 108.

1st: Find their factors:

Factors of 24:
2, 3, 4, 6, 8, **12**, 24

Factors of 72:
2, 3, 6, **12**, 36, 72

Factors of 108:
2, 3, 6, 9, **12**, 18, 36, 108

2nd: Choose the highest factor that is common to 24, 72 and 108:

The highest common factor is **12**

21 Find the highest common factor of:

a) 18, 24
b) 15, 6
c) 22, 33
d) 14, 6
e) 16, 24, 64
f) 45, 60, 15
g) 18, 36, 81
h) 36, 24, 30
i) 180, 45, 300
j) 160, 64, 32
k) 88, 66, 110
l) 150, 125, 50

22 We have two wires with lengths: 448 cm and 616 cm.
We want to cut them into pieces of equal length. Find the greatest possible length of the pieces.

23 Tim has 36 apples and Linda has 44 pears.
They have to put the fruits in bags with equal number of fruits; each bag containing only apples or only pears.
Find the largest possible number of fruits in each bag so that there is no remainder.

Multiples of 4:

0, 4, 8, 12, 16, 20, 24, etc.

$(4 \cdot 0 = 0\ ;\ 4 \cdot 1 = 4\ ;\ 4 \cdot 2 = 8\ ;$ etc.)

Multiples of 5:

0, 5, 10, 15, 20, 25, 30, etc.

$(5 \cdot 0 = 0\ ;\ 5 \cdot 1 = 5\ ;\ 5 \cdot 2 = 10\ ;$ etc.)

1.6 Multiples

The multiples of a number are the results of multiplying that number by other numbers.

For example: $4 \cdot 5 = 20$

20 is a multiple of 4

20 is also a multiple of 5

Find two multiples of 2:

To find multiples of 2, we multiply 2 by other numbers, for example 3 and 5

$2 \cdot 3 = 6$
Six is a multiple of two.

$2 \cdot 5 = 10$
Ten is a multiple of two.

Find the first five multiples of 2:

To find the first five multiples of 2, we multiply 2 by 0, 1, 2, 3 and 4

$2 \cdot 0 = 0$
$2 \cdot 1 = 2$
$2 \cdot 2 = 4$
$2 \cdot 3 = 6$
$2 \cdot 4 = 8$

The first five multiples of 2 are:
0, 2, 4, 6, 8

24 Find two multiples of these numbers:

a) 3 c) 45 e) 43
b) 8 d) 78 f) 246

25 Find the first five multiples of these numbers:

a) 2 c) 55
b) 7 d) 14

26 Find ten multiples of these numbers:

a) 1 c) 5
b) 4 d) 10

27 Which of the following numbers are multiples of 4?

a) 5, 7, 15, 8, 21, 24, 4, 34
b) 37, 44, 14, 3, 0

28 Alice has 23 €, Ben has 15 €, Lucy has 14 € and Tom has 26 €. If they put their money together, can they have it all in 20 € notes? Why?

29 We have boxes of 6 melons. We want to put as many boxes as possible in a car. Each melon weighs 3 Kg.
How many boxes can we put in the car if the maximum weight is 200 Kg?

Lowest common multiple (LCM)

The multiples of 2 are 2, 4, 6, 8, 10, **12**, 14, 16, 18, 20, 22, **24**, 26, etc.

The multiples of 3 are 3, 6, 9, **12**, 15, 18, 21, **24**, 27, 30, 33, etc.

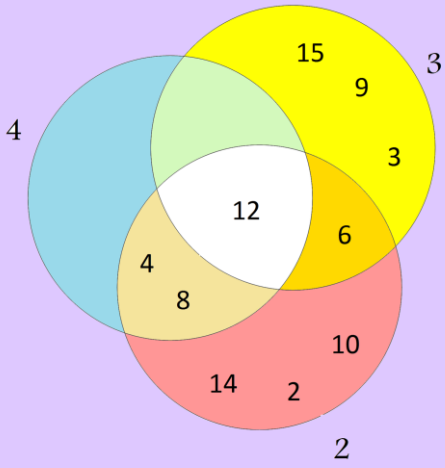
The multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, etc.

Numbers 12, 24, 36... are on the three lists, they are "common multiples".

Ignoring number 0, the lowest number which is on all lists is the "lowest common multiple" (LCM).

Number 12 is the lowest number which is on the three lists; it is the "lowest common multiple" (LCM).

The multiples in all groups are the common multiples.



The lowest number in all groups is 12, so 12 is the lowest common multiple of 2, 3 and 4

Find the lowest common multiple of: 8 and 10.

1st: obtain multiples by multiplying by 1, 2, 3...

Multiples of 8:

8, 16, 24, 32, 40, 48, 56...

Multiples of 10:

10, 20, 30, 40, 50, 60...

2nd: Choose the smallest number on both lists:

The lowest common multiple of 8 and 10 is **40**

30 Find the lowest common multiple of:

a) 2 and 11
b) 12 and 14
c) 5 and 7
d) 33 and 11
e) 24 and 36
f) 7 and 8

31 Find two numbers for which 7 is their LCM.

32 Athlete A runs around the track in 40 seconds. Athlete B runs around the track in 30 seconds.
How long would it take athlete B to pass athlete A?

33 A farmer must water the land every 12 days and go to the market every 14 days.
How often he must do both things on the same day?

34 A shop sells sandwiches in packs of 10.
Four friends want to buy the minimum number of packs so that they all get the same number of sandwiches.
How many packs must they buy?

Activities

35 Which of these numbers are integers?

a) 2/5
b) –6
c) 3
d) 2.6
f) 41
g) –6
h) 0
i) √2

36 Draw a number line and mark on it these integers: 4, 0, 2, –1, –4, 1

37 Draw a number line and mark on it these numbers, indicating if they are integers or not: 1, √5, √4, –3, –3/2, –1, 12/4, –6/2

38 Write the absolute value of these integers:

a) –2
b) 2
c) –12
d) 4
e) 42
f) –90

39 What integer values could be x?

a) $|x| = 3$
b) $|x| < 5$

40 Add and subtract these integers:

a) 2 – 1 + 3
b) 0 + 34 – 2
c) –2 – 6 + 36
d) –6 – 4 – 6 + 2
e) –2 – 8 + 8 – 8
f) 60 – 4 – 9 + 3 + 12
g) 7 – 1 – 62 + 15 + 78 – 2
h) 6 + 8 – 4 – 47 – 45 + 10 + 5

41 Multiply these integers:

a) 2 · 3
b) 4 · (–4)
c) (–2) · 5
d) (–2) · (–5)
e) (–37) · (–1)
f) (–15) · 2
g) 10 · (–15)
h) (–10) · 10

42 Divide these integers:

a) 8 / (–4)
b) 6 / (–3)
c) (–8) / 2
d) 2 / 2
e) (–12) / 4
f) (–21) / (–7)
g) (–30) / 3
h) (–81) / (–9)

43 Use the distributive property to solve:

a) 5 · (2 + 4)
b) –2 · (2 + 8)
c) 6 · (3 + 2)
d) 8 · (5 + 1)
e) 12 · (4 + 1)
f) 1 · (1 + 2)

44 Use the distributive property to solve:

a) 2 · (5 + 6)
b) 2 · (–4 + 1) + 2 · (1 + 7)
c) 6 · (5 + 2)
d) 8 · (5 – 1)
e) 5 · (3 – 1)
f) 2 · (3 + 5) + 4 · (5 – 3)
g) 6 · (5 + 2) – 2 · (3 – 1)
h) 4 · (6 – 5) + 3 · (8 – 1)

45 Use the distributive property and the order of operations to solve:

a) (6 / 2) + 2 · (5 – 6)
b) (12 – 4) / 2 + 6 · (1 + 7)
c) (4 · 5) / 10 – (5 · 2) + 6
d) 21 · (3 – 2 + 1) · (2 · 5)
e) [27/ (5 + 4)] – (3 – 1)
f) 75 – [20 / (4 · 5)] + [1 · (5 + 5)]

46 Solve:

a) 5 – [8 / (2 · 2)] – [2 · (6 + 12)]
b) [(20 + 5) / 25] + [56 · (4 – 3)]
c) (8 + 2) – [10 · (7 + 13)]
d) [5 · (2 – 1) + (5 + 5)] + 1 · (5 + 5)

47 Which are primes and which are composite numbers?

a) 6
b) 23
c) 83
d) 89
e) 51
f) 27
g) 120
h) 60

48 Factorize the following composite numbers:

a) 81
b) 22
c) 64
d) 128
e) 50
f) 86

49 Factorize the following composite numbers:

a) 300
b) 120
c) 220
d) 512
e) 129
f) 250
g) 500
h) 1,286

50 Find the highest common factor of:

a) 4 ; 6
b) 6 ; 18
c) 12 ; 20
d) 24 ; 64
e) 60 ; 15
f) 36 ; 81
g) 12 ; 200
h) 180 ; 45
i) 16 ; 20
j) 8 ; 10
k) 150 ; 100
l) 100 ; 125

51 Find the highest common factor of:

a) 8 ; 6 ; 10
b) 8 ; 12 ; 20
c) 21 ; 28 ; 14
d) 56 ; 24 ; 36
e) 15 ; 6 ; 18
f) 88 ; 22 ; 33
g) 14 ; 63 ; 21
h) 130 ; 30 ; 80
i) 90 ; 15 ; 25
j) 36 ; 216 ; 90
k) 100 ; 200 ; 150
l) 80 ; 40 ; 60
m) 75 ; 150 ; 50
n) 60 ; 90 ; 150

52 Find four multiples of these numbers:

a) 1
b) 3
c) 10
d) 53
e) 31
f) 45
g) 16
h) 12

53 Find the first five multiples of these numbers:

a) 1
b) 5
c) 6
d) 12
e) 11
f) 22
g) 400
h) 15
i) 100
j) 243
k) 220
l) 5,000

54 Find the first ten multiples of these numbers:

a) 9
b) 30
c) 15
d) 25
e) 105
f) 49

55 Find the lowest common multiple of:

a) 30 ; 75
b) 14 ; 21
c) 66 ; 44
d) 10 ; 15
e) 10 ; 14
f) 35 ; 40

56 Find the lowest common multiple of:

a) 3 ; 2 ; 5
b) 6 ; 5 ; 3
c) 4 ; 9 ; 2
d) 2 ; 3 ; 8
e) 14 ; 4 ; 56
f) 7 ; 8 ; 9

Word problems

57 What is an integer?

58 Are the absolute values of 5 and –5 the same? Why? Represent the absolute values on a number line.

59 Barry has 60 €. How much money will Barry have if he pays the telephone company 25 € and he also pays 15 € in the supermarket?

60 Sally has 187 € in the bank. Then she pays the telephone company 31 € and the electricity company 67 € and the gas company 86 € and 41 € in the supermarket, but she receives a payment in the bank for 200 €. How much money does she have now?

61 Sally owes 6 € to each of her 5 friends.
How much money does she owe?

62 Lilly bought 4 pairs of blue jeans at 28 € each. How much money did she spend?

63 A car travels 45 km from a school, and continues to travel for three hours on the same direction with a speed of 90 km per hour.
At what distance from the school is the car after the three hours?

64 A player receives 50 € for every correct answer and loses 45 € for every wrong answer.
In 30 questions, he fails 16, does he win any money?

65 In a winter day in Moscow the temperature was −4˚C at 9 p.m. At 9 a.m., the radio announced that the temperature was three times colder.
What was the temperature at 9 a.m.?

66 Linda bought 5 notebooks for 1.40 each and 3 boxes of pencils for 1.60 € each. How much did she spend?

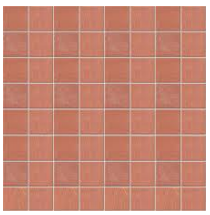
67 Andy borrows 500 € for a business.
The 1st year the profit is 120 €.
The 2nd year the profit is double the profit of the first year.
How much money he has at the end of the second year?

68 Mel borrows 100 € for a business.
The 1st year the profit is 200 €.
The 2nd year the profit is 42 €.
The 3rd year she loses 8 €.
The 4th year she halves the money that she had at the end of the third year.
The 5th year the profit is 26 €.
How much money she has at the end?

69 What is the difference between a prime number and a composite number?

70 Which of these numbers are factors of number 49?
2, 4, 7, 15, 24.

71 Ann wants to put 64 tiles on the floor forming a square or a rectangle.



In how many different ways can she put the tiles? (Number of rows and columns).

72 Now, Ann wants to put 40 equal boxes in piles of equal height.

How many different pile heights can she make?

73 What is the multiple of a number?

74 Which of the following numbers are multiples of 9?

a) 18, 28, 51, 54, 99, 180, 586.

b) 17, 27, 47, 74, 70, 140, 411.

75 What is the lowest common multiple of two numbers? And of three numbers? And of four numbers?

76 Athlete A runs around the track in 60 seconds. Athlete B runs around the track in 50 seconds.

How long would it take athlete B to pass athlete A?

77 Andrew travels to Dublin every 2 weeks. John travels to Dublin every 5 days.

On March 16th they are both in Dublin. When will they both be in Dublin again?

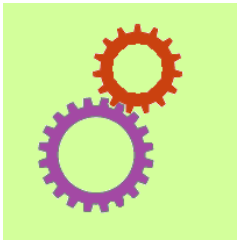
78 A traffic light goes green every 18 seconds, another every 24 seconds and another every minute.

They are all green at 12:00

a) When will they be all green again?

b) How many times will they be all green simultaneously after 12:00 and before 12:30?

79 Two gears start to turn together.



The small gear has 15 teeth. The big gear has 20 teeth.

a) How many teeth of each wheel have to pass before both wheels coincide again in the same position?

b) How many complete turns must each wheel turn to coincide again in the same position?

80 A frog jumps 120 cm each jump. Another frog jumps 100 cm each jump. If they start together, at what distance will they be together again?

81 www.esobook.es/b2

82 True or false? (If false explain why).

a) Integers contain the natural numbers, the negative numbers, and zero.

b) On a number line, "absolute value" is the distance between a number and its opposite.

c) $6 + 5 - 1 - 8 - 12 + 6 = 5$

d) $4 + 6 - 9 - 4 + 9 + 56 - 9 = 47$

e) $-7 + 6 - 5 + 5 + 45 - 5 - 1 = 38$

f) $9 + 56 - 57 - 2 - 15 + 46 = 37$

g) If we multiply a negative number by a positive number, the result is negative.

h) $5 \cdot 6 = -30$

i) $2 \cdot 6 - 4 + 5(3 + 1) = 12$

j) To multiply a number by the sum of others is the same thing as multiplying the number by each of the numbers in the addition.

k) $8(6 - 1) = 8 \cdot 6 - 8 \cdot (-1) = 40$

l) If there is a combination of operations, we add or subtract first.

m) The factors of a number are the numbers that add up exactly to the number.

n) Numbers 5, 8 and 12 are factors of number 40

o) The highest common factor of 54 is any number greater than 54

p) 55, 85 and 95 are multiples of 25

q) The lowest common multiple of two numbers is the lowest number which is a multiple of both numbers.