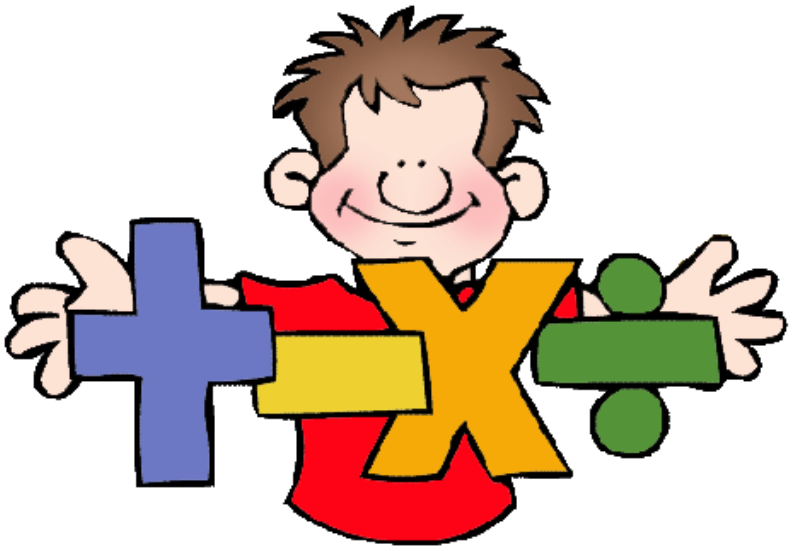
# Mathematics

# 3 ESO



esobook

# Contents

## Arithmetic


**1 REAL NUMBERS ..... 4**

1.1 FACTORS AND MULTIPLES ..... 5
1.2 FRACTIONS ..... 6
1.3 REAL ALGEBRAIC NUMBERS ..... 9
1.4 ROUNDING ..... 9
1.5 SCIENTIFIC NOTATION ..... 9

**2 POWERS AND ROOTS ..... 12**

2.1 OPERATIONS WITH POWERS ..... 13
2.2 RADICALS ..... 16

**3 SEQUENCES ..... 20**

3.1 TERMS ..... 20
3.2 ORDER ..... 21
3.3 ARITHMETIC PROGRESSIONS ..... 22
3.4 GEOMETRIC PROGRESSIONS ..... 23
3.5 SIMPLE INTEREST RATE ..... 24
3.6 COMPLEX INTEREST RATE ..... 25

**4 PROPORTIONALITY ..... 28**

4.1 DIRECT PROPORTIONALITY ..... 29
4.2 INVERSE PROPORTIONALITY ..... 30
4.3 COMPOUND PROPORTIONALITY ..... 31
4.4 APPLICATIONS OF PROPORTIONALITY ..... 32


## Algebra


**5 POLYNOMIALS ..... 36**

5.1 ADDITION ..... 38
5.2 MULTIPLICATION ..... 39
5.3 DIVISION ..... 40
5.4 IMPORTANT THEOREMS ..... 41
5.5 IMPORTANT EQUALITIES ..... 41

**6 EQUATIONS ..... 44**

6.1 SOLVING 1ST DEGREE EQUATIONS ..... 45
6.2 2ND DEGREE EQUATIONS ..... 46
6.3 SOLUTIONS ..... 48
6.4 SOLVING PROBLEMS WITH EQUATIONS ..... 50

**7 SYSTEMS OF LINEAR EQUATIONS ..... 54**

7.1 SOLUTIONS TO SYSTEMS OF LINEAR EQUATIONS 55
7.2 SOLVING SYSTEMS OF LINEAR EQUATIONS ..... 56
7.3 WHICH METHOD IS BEST? ..... 59

# Functions and graphs


**8 CHARACTERISTICS OF FUNCTIONS.....62**

8.1 How to express functions ..... 64
8.2 Continuity ..... 65
8.3 Asymptotes ..... 65
8.4 Trends ..... 66
8.5 Periodic functions ..... 66
8.6 Variation and critical points ..... 67
8.7 Transformations of functions ..... 68

**9 FUNCTIONS OF STRAIGHT LINES AND HYPERBOLAS..... 72**

9.1 Directly proportional functions ..... 73
9.2 Forms of writing functions ..... 74
9.3 Inversely proportional functions ..... 75


# Geometry


**10 TRANSFORMATIONS.....80**

10.1 Translation ..... 80
10.2 Rotation ..... 82
10.3 Reflection ..... 83
10.4 Enlargement ..... 83
10.5 Composition of transformations 84
10.6 Friezes and mosaics ..... 85

**11 PLANE GEOMETRY.....88**

11.1 Polygons ..... 88
11.2 Angles in a polygon ..... 91
11.3 Triangles ..... 92
11.4 The circumference ..... 94

**12 3-D SHAPES..... 98**

12.1 Polyhedrons ..... 98
12.2 Prisms ..... 100
12.3 Pyramids ..... 101
12.4 Solids of revolution ..... 102
12.5 Cylinders ..... 102
12.6 Cones ..... 102
12.7 Spheres ..... 104


# Statistics and probability


**13 STATISTICS.....108**

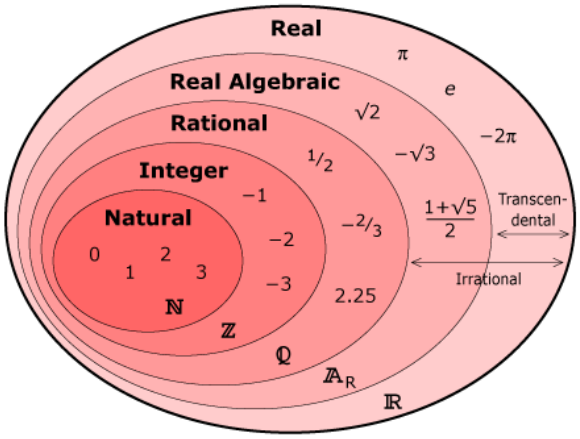
13.1 Frequency tables ..... 109
13.2 Charts ..... 110
13.3 Concentration parameters ..... 112
13.4 Dispersion parameters ..... 113

**14 PROBABILITY..... 116**

14.1 Experiments ..... 116
14.2 Events ..... 117
14.3 Frequency ..... 120
14.4 Law of large numbers ..... 120
14.5 Laplace's rule of succession ..... 121
14.6 Simple experiments ..... 122
14.7 Complex experiments ..... 123

# Chapter 1

# REAL NUMBERS




*Integers (Z)* are the natural numbers (N) and the negative numbers.

*Rational numbers (Q)* are the integers, the fractions, and the decimal numbers.

*Real Algebraic numbers ($A_R$)* are the rational numbers and the roots.

*Real numbers (R)* are the real algebraic numbers and the "transcendental" numbers: "π" and "e".

*Trascendental numbers* are real numbers, but they are not real algebraic numbers.

A real number is a value that represents a quantity:

- -8 (an integer)
- 5/3 or 2.3 (rational numbers that are not integers, given by a fraction or by a decimal)
- √2 (the square root of two, an algebraic number that is not rational, it has an infinite number of non-recurring decimals)
- π (3.1415926535..., a transcendental number)

We can represent real numbers along a number line, where the integers are equally spaced.

**1** Write ten rational numbers and five real algebraic numbers.

**2** Which of these "real algebraic" numbers are "rational"?

a) 1
b) 9/7
c) √40/2
d) 5/4
e) 8/√2
f) 5/15
g) 1/5
h) √73
i) 1.005

# 1.1 Factors and multiples

A number, **b**, is a factor of another number, **a**, if when we divide **a** over **b** the result is an integer.

**a** is a multiple of **b**.

Number 2 is always a factor of even numbers.

If the addition of the digits of a number is a multiple of 3, then number 3 is a factor of that number.

Number 5 is always a factor of numbers that have 0 or 5 as their last digit.

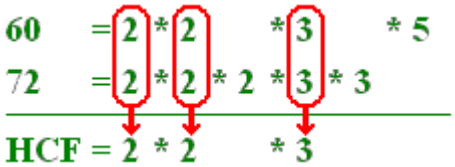
## Highest common factor (HCF)

The highest common factor of several numbers is the highest number that is a factor of each of those numbers.

For example, the common factors of 60 and 72 are:

2, 3, 4, 6, and 12.

So 12 is the highest common factor (HCF).



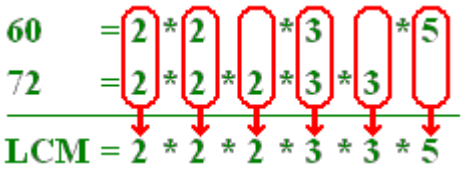

## Lowest common multiple (LCM)

The lowest common multiple of several numbers is the lowest number that is a multiple of each of those numbers.

For example, the multiples of 36 and 48 are:

144, 288, 432, …

So 144 is the "lowest common multiple" (LCM).




***3*** Which of these numbers: 75, 186, 612, 10, 5, 38, 81 are multiples of:

a) 2 b) 3 c) 6 d) 9

***4*** Find the HCF of:

a) 12 & 18 b) 25 & 35 c) 80 & 140

***5*** Find the LCM of:

a) 2 & 3 b) 5 & 7 c) 15 & 25

***6*** We must cut two bars in equal pieces. One bar is 64 mm long, the other bar is 80 mm long. How long can the pieces be?

***7*** John and Andrew are at the swimming pool on the first of June. John goes every 9 days. Andrew goes every 6 days. Give all the dates when they will meet again until the 15th of September.

# 1.2 Fractions

## Addition and subtraction of fractions

To add or subtract fractions:

*1st* Find the lowest common multiple of the denominators.

*2nd* Find equivalent fractions with the lowest common multiple as the denominator.

*3rd* Add or subtract the numerators.

$$\frac{2}{3}+\frac{1}{4}=\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$$

(12 is the LCM of 3 and 4)

## Multiplying fractions

- To multiply an integer by a fraction, multiply the integer by the numerator. The numerator is the result of the product and the denominator stays the same.

$$3\cdot\frac{7}{4}=\frac{3\cdot 7}{4}=\frac{21}{4}$$

- To multiply a fraction by another fraction:

  *1st* Multiply the numerators.

  *2nd* Multiply the denominators.

$$\frac{2}{5}\cdot\frac{7}{4}=\frac{2\cdot 7}{5\cdot 4}=\frac{14}{20}=\frac{7}{10}$$

## Dividing Fractions

- To divide a fraction by an integer, we multiply the integer by the denominator.
- To divide a fraction by another fraction:

  *1st* Multiply the numerator of the first fraction by the denominator of the second fraction. The result is the numerator of the resulting fraction.

  *2nd* Multiply the denominator of the first fraction by the numerator of the second fraction. The result is the denominator of the resulting fraction.

  Another option to divide fractions is to flip the second fraction, then multiply the fractions.

**8** Add:

a) $\frac{6}{5}+\frac{3}{2}$ b) $\frac{1}{3}+\frac{1}{2}$ c) $\frac{3}{2}+\frac{3}{4}+\frac{1}{2}$ d) $\frac{7}{5}+\frac{3}{4}+\frac{1}{6}$

**9** Subtract:

a) $\frac{12}{5}-\frac{5}{10}$ b) $\frac{15}{7}-\frac{2}{14}$ c) $\frac{325}{12}-\frac{10}{120}$ d) $\frac{22}{6}-\frac{18}{4}-\frac{9}{36}$

**10** Multiply:

a) $2\cdot\frac{6}{7}$ b) $\frac{6}{5}\cdot\frac{4}{5}$ c) (5/2) · (3/12) d) (1/2) · (3/4)

**11** Divide:

a) (6/7) / 5 b) (1/3) / (2/8) c) (2/21) / 4 d) (6/3) / (8/6)

**12** We have six bottles of 3/2 litres. How many litres do we have?

**13** 1/5 of the population is under 18 years of age. 9/10 of the population has dark eyes. What fraction of the population is under 18 and has dark eyes?

## Reduced Fraction

A fraction is “reduced” when we write it in its lowest terms.

We reduce a fraction by dividing numerator and denominator by their highest common factor.

For example: $\frac{2}{3}$ is the reduced fraction of $\frac{8}{12}$

## Converting a fraction into a decimal number

To convert a fraction into a decimal number, we divide the numerator by the denominator.

The decimal number can be:

- An “exact decimal”, for example $\frac{5}{4} = 1.25$
- A “pure recurring decimal” (or “repeating decimal”), for example $\frac{5}{3} = 1.666666666\ldots \quad 1.\hat{6}$
- A “mixed recurring decimal”, for example $\frac{284}{165} = 1.72121212121\ldots \quad 1.7\widehat{21}$

## Decimal fraction

The denominator of a decimal fraction is 1 followed by 0´s.

The decimal numbers from decimal fractions are always exact decimal numbers.

$$\frac{37}{10} = 3.7$$

## Ordinary fraction

The denominator of an ordinary fraction is not 1 followed by 0´s.

The decimal numbers from ordinary fractions are sometimes exact decimal numbers and sometimes recurring decimal numbers (pure or mixed).

**14** Reduce these fractions:

a) 20/360 c) 10/1000 e) 10/155

b) 34/8 d) 150/30 f) 171/45

**15** Convert into decimal numbers and say if the decimal number is “exact”, “pure recurring” or “mixed recurring":

a) 4/6 c) 36/10 e) 5/12

b) 2/9 d) 4/3 f) 347/1000

**16** Which of these fractions are decimal fractions and which are ordinary fractions?

a) 32/7 c) 36/100 e) 8/1000

b) 9/10 d) 24/3 f) 35/2

## Convert an exact decimal number into a fraction

To change an exact decimal number to a common fraction:

- Numerator: the number without the decimal point.
- Denominator: 1 followed by as many 0´s as decimal places.
- Simplify (reduce) the fraction.

$$5.25 = \frac{525}{100} = \frac{21}{4}$$

two decimal places, so two zeros

## Convert a recurring decimal number into a fraction

To change a "recurring decimal number" to a common fraction:

- Numerator: the number without the decimal point minus the "whole" part (the non-decimal part).
- Denominator: as many nines as recurring decimal digits.
- Simplify the fraction.

$$2.\widehat{6} = \frac{26 - 2}{9} = \frac{24}{9} = \frac{8}{3}$$

one recurring digit, so one nine

## Convert a mixed recurring decimal number into a fraction

To change a decimal number to a common fraction:

- Numerator: the number without the decimal point minus the "whole" part (the non-decimal part) followed by the non-recurring decimals.
- Denominator: as many nines as recurring decimal digits, followed by as many zeros as non-recurring decimal digits.
- Simplify the fraction.

$$2.7\widehat{63} = \frac{2763 - 27}{990} = \frac{2736}{990} = \frac{152}{55}$$

two recurring digits, so two nines
one non-recurring digit, so one zero

Convert 0.25 to fraction:

1st We write the numerator as the number without the dot: 25

2nd We write the "unit multiplier": 100

3rd We can reduce (simplify) 25/100, its simplest equivalent fraction is: 1/4

**17** Convert these exact decimal numbers to common fractions:

a) 2.4 c) 23.02 e) 100.8
b) 8.45 d) 40.25 f) 246.4556

**18** Convert these purely recurring decimal numbers to common fractions:

a) $6.\hat{4}$ c) $5.\widehat{42}$ e) $0.\widehat{25}$
b) $0.\hat{4}$ d) $1.\widehat{38}$ f) $0.\widehat{05}$

**19** Convert these mixed recurring decimal numbers to common fractions:

a) $6.1\hat{3}$ c) $5.8\widehat{41}$ e) $56.95\hat{9}$
b) $2.0\hat{3}$ d) $45.77\widehat{46}$ f) $8.33\hat{4}$

## 1.3 Real algebraic numbers

Real Algebraic numbers include the natural numbers, the integers, the rational numbers and the numbers that we can't write as a fraction.

If we write them as a decimal number, they have infinite non-recurring decimal places.

For example, the square root of number 2 is
$\sqrt{2} = 1.414213562\ldots$

1.653 is a rational number because it has a finite number of decimal places.

1.55555555… is a rational number because it is a recurring decimal number.

3.6564797304… is a real algebraic number because it has an infinite number of non periodic decimal places.

## 1.4 Rounding

To round a number is to replace it by another number that is shorter or simpler. The "rounded number" is an approximation of the original number.

For example, if we round 43.771 to one decimal place, the rounded number is 43.8

If we round 683,475 to the thousands, the rounded number is 683,000

### Rounding error

When we round we get a "round-off" error. The error is the result of subtracting the exact number from the round number.

For example, rounding 43.77163 to 43.8 gives an error of:

43.77163 - 43.8 = -0.02837  the error is -0.02837

## 1.5 Scientific notation (or "standard index form")

Scientific notation is a way of writing numbers that are very large or small.

In scientific notation we write numbers like this: $a \cdot 10^b$

For example: 2 millions (2,000,000) is $2 \cdot 10^6$ "two times ten to the power of six".

Five thousandths (0.005) is $5 \cdot 10^{-3}$ "five times ten to the power of negative three".

***20*** Which of these numbers are real algebraic? Why?

a) 1/3  b) 6  c) π  d) √7

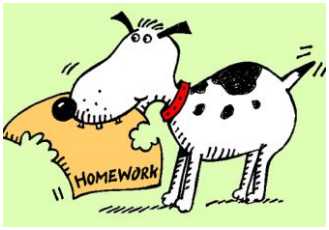
***21*** Round these numbers to one decimal place and calculate the rounding error:

a) 6.978  b) 1.1252  c) 0.0438

***22*** Write in scientific notation:

a) 6,000,000  b) 1,600,000,000  c) 0.006

# Activities




**23** Classify these numbers as: natural, integers, rational, real algebraic or transcendental:

a) 1/4
b) π
c) 8
d) √6
e) -12
f) e
g) 4
h) √73
i) 5/15
j) (1+√2)/5

**24** Which of these numbers: 75, 186, 612, 11, 25, 38, 93 are multiples of:

a) 5
b) 306
c) 25
d) 31

**25** Find the HCF of:

a) 27 & 18
b) 8& 24
c) 80 & 140
d) 21 & 35

**26** Find the LCM of:

a) 12 & 15
b) 8 & 7
c) 345 & 15
d) 26 & 54

**27** Andy wants to put 9 litres of olive oil and 24 litres of sunflower oil in bottles (without mixing them).
He wants to use only one size of bottles.
What is the largest size of bottles he can use?

**28** Racing car A goes around the track in 220 seconds. Racing car B goes around the track in 180 seconds. If they start together, after how many seconds will car B pass car A?

**29** Add:

a) 1/5 + 6/8
b) 1/3 + 1/4
c) 2/7 + 1/3 + 2/8
d) 1/5 + 1/3 + 1/2

**30** Add:

a) 1/3 + 2/5
b) 5/3 + 7/4
c) 4/6 + 6/3 + 1/3
d) 1/3 + 2/3 + 3/9

**31** Subtract:

a) 1/2 – 4/6
b) 1/5 – 1/10
c) 1/4 - 2/9 - 2/9
d) 7/2 – 5/7 – 1

**32** Subtract:

a) 3/2 – 5/2
b) 6/5 – 2/5
c) 5/3 - 3/6 - 2/2
d) 6/1 – 4 – 3/4

**33** Calculate:

a) 1/2 + 1/3 – 4/3
b) 1/4 - 2/6 + 6/8 - 2/7
c) 4/5 + 2/12 + 1/3
d) 4/2 – 5/4 + 1 + 1/6 - 2/5

**34** Tesa and Andrew buy a pizza, Tesa eats 2/8 of the pizza and Andrew eats 4/6 of the pizza. What fraction of the pizza is left?

**35** Multiply:

a) 6 • (3/5)
b) 5 • (14/21)
c) 14 • (3/2)
d) 8 • (20/12)

**36** Andrew has oranges, lemons and mandarins. In total, he has 2500 pieces of fruit. 1/5 are oranges and 3/10 are lemons.

a) What is the fraction of mandarins?
b) How many mandarins does Andrew have?

**37** Alison eats one fourth of the pizza. John eats three times more than her. How much pizza does John eat?

**38** A train can transport 448 people. If it is 1/7 full, how many people are on the train?

**39** Sally earns 1710 €. If she spends 2/5 on house expenses and 290 € on food, how much money does she still have?

**40** Sam has 3/2 hectares of land. Sally has ten times more land than Sam. How much land does Sally have?

**41** Multiply:

a) (4/5) • (3/5)
b) (2/3) • (3/2)
c) (3/2) • (14/21)
d) $\frac{1}{2} \cdot \frac{20}{12} \cdot \frac{1}{2}$

**42** Pirates Rachel and Steve divide the treasure, so each has $\frac{1}{2}$. Rachel spends $\frac{3}{5}$ of her part on a new ship. What fraction of the treasure does the ship cost?

**43** Divide:

a) (1/2) / 2
b) (7/2) / 3
c) (3/8) / 3
d) 5/3) / 6

**44** Johnny owns half the land of Texas. When he dies, each of his three children inherit one third of his land. What fraction of the land of Texas does each child own?

**45** Divide:

a) (5/2) / (8/6)
b) (6/3) / (1/4)
c) (5/2) / (1/4)
d) (7/3) / (8/6)

**46** Reduce these fractions:

a) 24/360
b) 100/150
c) 100/10
d) 35/7

**47** Convert into decimal numbers:

a) 1/10
b) 5/20
c) 1/8
d) 45/10
e) 35/73
f) 354/26

**48** Which of these fractions are decimal fractions and which are ordinary fractions?

a) 32/62
b) 9/8
c) 2/100
d) 4/9
e) 4/10
f) 567/1000

**49** Convert to fractions:

a) 1.2
b) 2.61
c) 21.763
d) 230.605

**50** Convert to fractions:

a) $5.\widehat{3}$
b) $6.\widehat{3}$
c) $0.\widehat{56}$
d) $12.\widehat{07}$

**51** Convert to fractions:

a) $7.0\widehat{6}$
b) $5.\widehat{284}$

**52** Which of these fractions: 1/7 , 34/22 , 6/11 , 1/4 is:

a) The conversion into a fraction of decimal number 0.25?
b) A common fraction without simplifying?

**53** Classify these decimal numbers as exact, pure recurring or mixed recurring:

a) 0.4
b) 5.2323232323…
c) 9.3422222222…
d) 7.3452
e) 234.493522522522522522…
f) 93.93939393939393…

**54** Which of these numbers are real algebraic?

a) $\sqrt{3}$
b) e
c) 0
d) -8
e) 1/3
f) π

**55** Round these numbers to two decimal places and calculate the rounding error:

a) 24.5978
b) 6.7128
c) 8765.74529
d) 23.5552

**56** Write in scientific notation:

a) 2,000
b) 300,000
c) 1,000,000,000
d) 500

**57** Write in scientific notation:

c) 0.02
d) 0.0006
c) 0.0000001
d) 0.00005

**58** www.esobook.es/c1

**59** True or false?

a) A natural number may not be rational.
b) A rational number may not be natural.
c) If “a” is a multiple of “b”, then “b” is a factor of “a”.
d) A multiple of a number is always smaller than the number.
e) The decimal numbers from decimal fractions are exact decimal numbers.
f) 6,000,000,000 in scientific notation is $6 \cdot 10^9$